BINARY LAMINAR BOUNDARY LAYER ON A VERTICAL SURFACE IN THE PRESENCE OF COMBINED FREE AND FORCED CONVECTION

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Heat and mass transfer at a vertical surface is examined in the case of combined free and forced convection. The boundary layer equations, transformed to ordinary differential equations, contain a parameter that determines the effect of free convection on the forced motion. Criteria are offered for differentiating the free-convection, forced-convection, and combined regimes.

NOTATION

x, y—coordinates
u, v—velocity components
g—acceleration of gravity
T—temperature
ν—kinematic viscosity
β—coefficient of thermal expansion
a—thermal diffusivity
ρ_1—partial vapor density
D—diffusion coefficient
W_2—mass velocity of air
η—independent variable
τ_w—shear stress at wall
λ—thermal conductivity
r—latent heat of phase transition
θ, φ—dimensionless temperature and partial vapor density
m*—the complex $(m_{1\infty}-m_{1w})/(1-m_{1w})$,
c_p—specific heat at constant pressure
G—Grashof number
R—Reynolds number
P—Prandtl number
S—Schmidt number

$$G=\frac{g\beta(T_w-T_\infty)x^3}{\nu^2},\quad R=\frac{U_\infty x}{\nu},\quad P=\frac{\nu}{a},\quad S=\frac{\nu}{D}.$$

The subscript w denotes values at the surface, ∞ denotes values remote from the surface, 1 is for vapor, 2 is for air.

Consider (Fig. 1) a vertical plate with constant temperature T_w and partial density of component 1 ρ_{1w} in a binary-mixture flow moving at velocity U_∞ along the plate in the direction of action of the lift forces.

The differential equations of the laminar incompressible boundary layer, neglecting viscous dissipation and without allowance for thermal diffusion and diffusional heat conduction and on the assumption that $c_{p1}=c_{p2}$ and that the physical parameters are constant, are written in the form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty) \tag{1}$$
$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2},\quad u\frac{\partial \rho_1}{\partial x}+v\frac{\partial \rho_1}{\partial y}=D\frac{\partial^2 \rho_1}{\partial y^2}$$

with the following boundary conditions

$$u=0,\quad v=v_w,\quad T=T_w,\ \rho_1=\rho_{1w}\quad \text{at}\quad y=0$$
$$u=U_\infty,\ T=T_\infty,\ \rho_1=\rho_{1\infty}\ \text{at}\ y=\infty. \tag{2}$$

We also assume that the velocity of the condensing fluid at the wall and its thermal resistance are negligibly small in comparison with the freestream velocity and the thermal resistance of the boundary layer. Moreover, we have omitted from the equation of motion the term representing the lift due to the concentration difference.

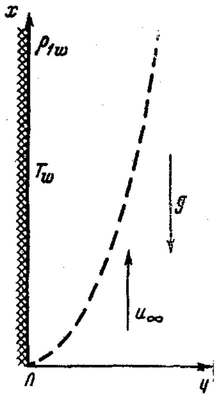

Fig. 1

We reduce system (1) to ordinary differential equations by introducing the independent variable η and the stream function ψ:

$$\eta=y\left(\frac{U_\infty}{\nu x}\right)^{1/2},\quad \Psi=(U_\infty \nu x)^{1/2}f(\eta)\quad \left(u=\frac{\partial\Psi}{\partial y},\quad v=-\frac{\partial\Psi}{\partial x}\right). \tag{3}$$

In the new variables we have

$$u=U_\infty f'(\eta),\quad v=-\frac{1}{2}\left(\frac{U_\infty\nu}{x}\right)^{1/2}[f(\eta)-\eta f'(\eta)]. \tag{4}$$

Then instead of system (1) we obtain

$$\begin{gathered} f'''(\eta)+\frac{1}{2}f(\eta)f''(\eta)+\frac{G}{R^2}\theta(\eta)=0\\ \theta''(\eta)+{}^1\!/_2\,P\,f(\eta)\,\theta'(\eta)=0,\qquad \varphi''(\eta)+{}^1\!/_2 Sf(\eta)\,\varphi'(\eta)=0 \end{gathered} \tag{5}$$

where

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \varphi(\eta)=\frac{\rho_1-\rho_{1\infty}}{\rho_{1w}-\rho_{1\infty}}.$$

The transformed equations contain the parameter $G/R^2 = A$, which does not depend on η. When this parameter is equal to zero, Eqs. (5) are converted into the equations for forced convection; at large values of A the free convection process will obviously predominate. A prime denotes differentiation with respect to η.

In the new variables boundary conditions (2) for system (5) take the form:

$$f'(0)=0,\quad f_w=\text{const},\quad \theta=1,\quad \varphi=1\quad \text{at}\quad \eta=0$$
$$f'(\infty)=1,\quad \theta=0,\quad \varphi=0\quad \text{at}\quad \eta=\infty. \tag{6}$$

The boundary condition f_w = const implies that

$$v_w = -\frac{1}{2} f_w \left(\frac{U_\infty \nu}{x}\right)^{1/2}, \text{ т. е. } v_w \sim x^{-1/2}.$$

This limitation does not affect the generality of the conclusions reached in this paper. As shown in [1,2], the law of variation of v_W for free and forced convection has a relatively weak effect on the variation of the characteristics of the boundary layer and the local heat transfer coefficient. In most cases the law $v_w \sim x^{-1/2}$ is consistent with the condition of constant temperature and mass content at the surface. The mass velocities of air and vapor at the surface of the plate are

$$W_{2w} = -D\left(\frac{\partial \rho_2}{\partial y}\right)_{y=0} - v_w \rho_{2w} = 0$$
$$W_{1w} = -D\left(\frac{\partial \rho_1}{\partial y}\right)_{y=0} - v_w \rho_{1w} \tag{7}$$

which gives

$$v_w = -\frac{D}{\rho_{2w}}\left(\frac{\partial \rho_2}{\partial y}\right)_{y=0} = -\frac{D}{1 - m_{1w}}\left(\frac{\partial m_1}{\partial y}\right)_{y=0} \tag{8}$$

since

$$m_1 + m_2 = 1, \quad m_1 = \rho_1 / \rho, \quad m_2 = \rho_2 / \rho$$

and hence

$$W_{1w} = -\rho D \frac{1}{1 - m_{1w}}\left(\frac{\partial m_1}{\partial y}\right)_{y=0}. \tag{9}$$

From relation (3), (6), (8), and (9) we find that

$$f_w = -\frac{2}{S} \frac{m_{1\infty} - m_{1w}}{1 - m_{1w}} \varphi'(0) \tag{10}$$

where m_{1W} is the vapor mass content at the surface.

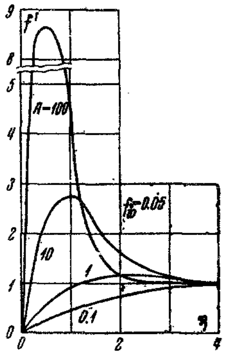

Fig. 2

Nonlinear system (5) with boundary conditions (6) was solved on an M-20 computer by iteration using the pivotal method [3,4]. As the zero-order approximation for $f(\eta)$ we took the Blasius solution [5].

As a result of these calculations we obtained the velocity and temperature profiles and the distribution of the partial density of component 1 in the boundary layer (Figs. 2,3,4) for combined free and forced convection for the numbers P = 0.72, S = 0.6 and a parameter A equal to 0.1, 1, 10, 100 at $f_W = 0.05$. By way of example, Fig. 4 shows the effect of the mass flux on the temperature profile for A = 0.1. The solid lines in Fig. 3 represent the density distribution of component 1; the dashed lines the temperature distribution. The problem for the case $f_W = 0$, but without allowance for phase transitions, has been solved by Szewczyk [6]. In this case our data coincide with the results of [6].

The local skin friction at the wall is given by the expression

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} - \rho v_w U_\infty . \tag{11}$$

From relation (4) we find that

$$\left(\frac{\partial u}{\partial y} \right)_{y=0} = U_\infty \left(\frac{U_\infty}{\nu x} \right)^{1/2} f''(0).$$

Then from (11) we have

$$\tau_w = \mu \frac{U_\infty^2}{\nu^2} \frac{f''(0)}{R^{1/2}} + \frac{1}{2} \rho U_\infty^2 \frac{f_w(0)}{R^{1/2}}$$

or in dimensionless form

$$\frac{\tau_w}{\rho U_\infty^2} = \frac{f''(0)}{R^{1/2}} + \frac{1}{2} \frac{f_w(0)}{R^{1/2}} . \tag{12}$$

The total heat flow (with allowance for the heat of phase transition) through the wall is calculated from the equation

$$q_w^* = -\lambda \left(\frac{\partial T}{\partial y} \right)_{y=0} \pm \frac{r \rho D}{1 - m_{1w}} \left(\frac{\partial m_1}{\partial y} \right)_{y=0} \tag{13}$$

taking the plus sign for evaporation and the minus sign for condensation.

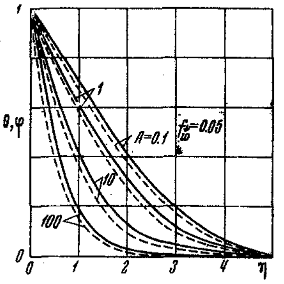

Fig. 3

Then the heat transfer coefficient (with allowance for the heat of phase transition) is

$$\alpha^* = \frac{q_w^*}{T_w - T_\infty} = -\lambda R^{1/2} x^{-1} \left\{ \theta'(0) \pm \frac{P}{S} \frac{r m^*}{c_p (T_w - T_\infty)} \varphi'(0) \right\} \tag{14}$$

or

$$N = \frac{\alpha^* x}{\lambda} = -R^{1/2} \left\{ \theta'(0) \pm \frac{P}{S} \frac{r m^*}{c_p (T_w - T_\infty)} \varphi'(0) \right\} . \tag{15}$$

The first term in the braces gives the convective component of the heat flux. Similarly, for the mass flux

$$j_1 = -\rho D \left(\frac{\partial m_1}{\partial y} \right)_{y=0} = -\rho D \left(\frac{U_\infty}{\nu x} \right)^{1/2} (m_{1w} - m_{1\infty}) \varphi'(0)$$

or the mass transfer coefficient

$$\alpha_m = \frac{j_1}{\rho (m_{1w} - m_{1\infty})} = -D R^{1/2} x^{-1} \varphi'(0) \tag{16}$$

and hence the Nusselt number for mass transfer will be

$$N_D = \frac{\alpha_m x}{D} = -R^{1/2}\varphi'(0). \tag{17}$$

The conditions under which the heat transfer process may be regarded either as an exclusively free-convection or exclusively forced-convection flow can be determined by comparing the numerical calculation with the results of a calculation of the heat transfer for purely forced and purely free convection at P = 0.72 in accordance with the equations

$$\frac{N}{R^{1/2}} = 0.297, \quad \frac{N}{R^{1/2}} = -\frac{\theta'(0)}{\sqrt{2}}\left(\frac{G}{R^2}\right)^{1/4} \tag{18}$$

The first of relations (18) was obtained from the results of this paper and coincides with the data of other authors [1], while the second of these relations was obtained from the results of [7].

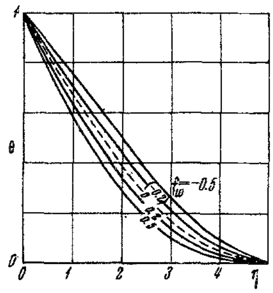

Fig. 4

If it is assumed [8] that the heat transfer for purely forced or purely free convection differs from (18) by not more than 5%, then the boundaries of these flows can be determined from the conditions

$0 < A < 0.095$ – forced convection
$0.095 < A < 16$ – combined convection
$16 < A$ – free convection

Similarly, from the known expressions [5,7]

$$\tfrac{1}{2}C_f R^{1/2} = 0.328, \quad \tfrac{1}{2}C_f R^{1/2} = \sqrt{2}f''(0)\left(\frac{G}{R^2}\right)^{3/4} \tag{19}$$

it is possible to obtain the boundaries of the flow regimes for calculating the skin friction

$0 < A < 0.015$ – forced convection
$0.015 < A < 16$ – combined convection
$16 < A$ – free convection.

The above conclusions relating to the limits of the three characteristic regimes also apply when $f_w \neq 0$.

Having studied the dynamic and thermal characteristics of the boundary layer we now turn to a consideration of the problem of mass transfer in the presence of combined free and forced convection. The table presents $\theta'(0)$, $\varphi'(0)$ and m^* for various values of the parameter A at P = 0.72, S = 0.9 and 0.6 for both condensation and evaporation. For $f_w > 0$ we have condensation, and for $f_w < 0$ evaporation from the vertical surface.

Calculations were also made for the case in which the forced and free convection are opposite in direction. In this case in boundary conditions (6) with $\eta = \infty$ we have $f'(\infty) = -1$, but the other conditions do not change. The system of equations (5) also remains unchanged.

Expressions (12), (15), and (17) for the skin friction and local Nusselt numbers remain valid for the process characterized by opposite directions of the free and forced convection.

Free and Forced Convection Coincide in Direction

f_w	$\theta'(0)$ $P=0.72$	$S=0.6$ $\varphi'(0)$	$S=0.6$ m^*	$S=0.9$ $\varphi'(0)$	$S=0.9$ m^*
		$A=0.1$			
0.06	—0.3295	—0.3105	0.0579	—0.3561	0.0758
0.04	—0.3246	—0.3064	0.0391	—0.3499	0.0514
0.02	—0.3198	—0.3024	0.0198	—0.3439	0.0263
0.0	—0.3149	—0.2984	0.0	—0.3380	0.0
—0.02	—0.3102	—0.2944	—0.0203	—0.3319	—0.0271
—0.04	—0.3454	—0.2904	—0.0413	—0.3259	—0.0552
—0.06	—0.3006	—0.2864	—0.0628	—0.3201	—0.0343
		$A=1$			
0.06	—0.4125	—0.3835	0.0469	—0.4518	0.0597
0.04	—0.4082	—0.3799	0.0316	—0.4464	0.0403
0.02	—0.4039	—0.3764	0.0159	—0.4409	0.0204
0.0	—0.3998	—0.3730	0.0	—0.4359	0.0
—0.02	—0.3956	—0.3696	—0.0162	—0.4306	—0.0209
—0.04	—0.3913	—0.3660	—0.0361	—0.4252	—0.0423
—0.06	—0.3871	—0.3625	—0.0480	—0.4199	—0.0643
		$A=10$			
0.06	—0.6393	—0.5881	0.0306	—0.7064	0.0382
0.04	—0.6354	—0.5849	0.0205	—0.7014	0.0256
0.02	—0.6315	—0.5815	0.0103	—0.6964	0.0129
0.0	—0.6277	—0.5786	0.0	—0.6916	0.0
—0.02	—0.6235	—0.5754	—0.0104	—0.6866	—0.0132
—0.04	—0.6201	—0.5722	—0.0209	—0.6817	—0,0264
—0.06	—0.6162	—0.5681	—0.0316	—0.6768	—0.0399
		$A=100$			
0.06	—1.0756	—0.9376	0.0182	—1.1889	0.0227
0.04	—1.0716	—0.9344	0.0122	—1.1837	0.0152
0.02	—1.0679	—0.9313	0.0061	—1.1788	0.0076
0.0	—1.0641	—0.9783	0.0	—1.1741	0.0
—0.02	—1.0603	—0.9751	—0.0061	—1.1692	—0.0077
—0.04	—1.0564	—0.9720	—0.0123	—1.1643	—0.0154
—0.06	—1.0526	—0.9689	—0.0186	—1.1594	—0.0233

Free and Forced Convection Opposite in Direction

f_w	$\theta'(0)$ $P=0.72$	$S=0.6$ $\varphi'(0)$	$S=0.6$ m^*	$S=0.9$ $\varphi'(0)$	$S=0.9$ m^*
		$A=0.25$			
0.06	—0.2162	—0.2138	0.0341	—0.2199	0.1224
0.04	—0.2133	—0.2114	0.0567	—0.2162	0.0933
0.02	—0.2104	—0.2090	0.0287	—0.2124	0.0424
0.0	—0.2076	—0.2067	0.0	—0.2090	0.0
—0.02	—0.2047	—0.2043	—0.0294	—0.2053	—0.0434
—0.04	—0.2019	—0.2019	—0.0594	—0.2018	—0.0891
—0.06	—0.1991	—0.1996	—0.0902	—0.1982	—0.1361
		$A=1$			
0.06	—0.3087	—0.2904	0.0320	—0.3359	0.0804
0.04	—0.3054	—0.2878	0.0417	—0.3317	0.0543
0.02	—0.3021	—0.2852	0.0211	—0.3275	0.0275
0	—0.2939	—0.2825	0.0	—0.3235	0.0
—0.02	—0.2959	—0.2801	—0.0214	—0.3193	—0.0282
—0.04	—0.2927	—0.2775	—0.0433	—0.3152	—0.0571
—0.06	—0.2895	—0.2749	—0.0655	—0.3111	—0.0868
		$A=10$			
0.06	—0.5849	—0.5311	0.0339	—0.6562	0.0412
0.04	—0.5814	—0.5282	0.0227	—0.6515	0.0276
0.02	—0.5778	—0.5253	0.0114	—0.6468	0.0139
0.0	—0.5742	—0.5224	0.0	—0.6421	0.0
—0.02	—0.5706	—0.5195	—0.0115	—0.6375	—0.0141
—0.04	—0.5671	—0.5167	—0.0232	—0.6328	—0.0284
—0.06	—0.5635	—0.5138	—0.0350	—0.6282	—0.0429
		$A=100$			
0.06	—1.0470	—0.9552	0.0188	—1.1636	0.0232
0.04	—1.0432	—0.9528	0.0126	—1.1587	0.0155
0.02	—1.0395	—0.9492	0.0063	—1.1539	0.0078
0	—1.0358	—0.9462	0.0	—1.1492	0.0
—0.02	—1.0332	—0.9432	—0.0064	—1.1445	—0.0079
—0.04	—1.0284	—0.9402	—0.0128	—1.1397	—0.0158
—0.06	—1.0247	—0.9372	—0.0192	—1.1348	—0.0240

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